The T-10 tokamak experimental data on the rotation of $m = 2$ mode with changing amplitude in the presence of the externally applied static Resonant Magnetic Perturbation (RMP) are presented. In addition to the well-known effect of the mode locking (termination of mode rotation) in the case of sufficiently large mode amplitude, a locking of the mode with significantly smaller amplitude is observed. If the amplitude exceeds a certain threshold level, the mode is in the state of an irregular rotation. With a decrease of the amplitude lower than this threshold value, the mode rotation stops and recovers when the amplitude surpasses this level again. The experimental data agree with the result of numerical simulation with the TEAR code utilizing the model of the non-linear Rutherford tearing mode in rotating plasma in the presence of the RMP. The locking of the small magnetic islands is attributed to the asymmetric effect of the RMP on the tearing mode stability index $\Delta'$, resulting in deviations of the magnetic island rotation from the rotation of the plasma layer in the vicinity of the rational magnetic surface. In this paper, the attention is paid to the dynamics of the tearing mode with relatively small amplitude. This mode can participate in the process of RMP penetration into plasma and play an essential role in the initial development of the neoclassical tearing mode.

**Key words:** tokamak, plasma, tearing mode, error field, magnetic island locking.

**INTRODUCTION**

In the publications on nuclear fusion, a considerable attention is paid to the experimental and theoretical research of the conventional and neoclassical tearing mode dynamics [1, 2]. The magnetic islands arising under development of the tearing mode can occupy a significant part of the plasma volume and deteriorate the plasma confinement and thermal insulation. Under some conditions, large magnetic islands can trigger the disruptive instability. The attention is also paid to the research of relatively small magnetic islands because they play the role of seed islands for the neoclassical tearing mode [3, 4]. Besides that the magnetic islands participate in the process of penetration into plasma of the externally applied helical magnetic field [5—7].

The magnetic island structures in the tokamak plasma are usually rotating, the rotation being irregular. The rotation irregularity appears as cyclical variations of the mode instantaneous angular velocity [8—10]. In appropriate conditions, the increase of the rotation irregularity is ultimately followed by the termination of the mode rotation, i.e. mode locking. It is well known that the magnetic island rotation irregularity and locking can be attributed to the effect of the externally applied Resonant Magnetic Perturbation [5, 6, 11, 12]. The RMP has the same poloidal, $m$, and toroidal, $n$, wave numbers as the mode under consideration. The static RMP (the so-called Error Field) is present in all tokamaks due to inevitable misalignments of the magnetic field coils. In some experiments, the RMP generated with special coils is used for the control of plasma behavior.

According to [13], a non-monotonic dependence of the rotation irregularity on the mode amplitude is observed for broad variation of the amplitude. The rotation irregularity increases in both cases of sufficiently big
and small amplitudes. In the case of sufficiently large magnetic islands the rotation irregularity and locking are attributed to the effect of the RMP on the rotation of resonant plasma layer occupied by magnetic islands [6]. The rotation irregularity takes place due to cyclical variation of the electromagnetic torque applied to the resonant layer in the process of magnetic island rotation. This torque amplitude increases with the growth of the magnetic islands causing the increase of the rotation irregularity up to the termination of mode rotation.

In the case of sufficiently small magnetic islands, the rotation irregularity is attributed to the effect of the RMP on the tearing mode stability index \( \Delta ' \) (see [13]). This effect depends on the instantaneous angular shift between the magnetic island structure and RMP. It results in cyclical variations of the mode velocity with respect to the velocity of the resonant plasma layer along with the rotation of the magnetic islands. The rotation irregularity increases with the decrease of the mode amplitude because the dependence of \( \Delta ' \) on RMP becomes stronger with the reduction of the magnetic island width.

Since the rotation irregularity increases with the reduction of the island width in the case of small magnetic islands, one can expect that the mode rotation will stop at sufficiently small mode amplitude. An experimental observation confirming this assumption is presented in this paper. We consider the magnetic islands that are small in comparison with large islands locked due to the well-known effect of the Error Field on the rotation of the plasma resonant layer [6]. In our experiment the large magnetic island locking followed by minor disruptions took place at mode amplitude higher than 10\(^{-3}\) T.

The experiment is numerically modeled with the TEAR code [13, 14] based on the model which takes into account the dependence of the tearing mode stability index \( \Delta ' \) on the angular position of the mode with respect to the RMP. Unlike [6], deviations of magnetic island rotation velocity from the velocity of the resonant plasma layer are not neglected in this model. The assumption that the island does not ‘slip’ through the plasma, used in [6], is valid in the case of sufficiently large magnetic islands and high plasma conductivity. In the opposite case when the islands and the plasma conductivity are sufficiently small, the difference between the rotation velocities of islands and of the plasma resonant layer should be taken into account.

**EXPERIMENTAL ARRANGEMENT AND PROCESSING OF THE MHD DATA**

The T-10 facility is a tokamak with a circular plasma cross section. The major and minor radii of the vacuum vessel are \( R = 1.5 \) m and \( r_{VV} = 0.41 \) m, respectively. The minor radius of the plasma was determined by the movable limiter position \( a = 0.27 \) m. An ohmic tokamak regime with low electron temperature and density characterized by the \( m = 2 \) mode with non-monotonically varying amplitude was chosen for the experiment, because this regime is the most convenient to observe the effect under consideration. The waveforms of the main discharge parameters are shown in Fig. 1. In this experiment, we have not used any artificially applied RMP. Only the intrinsic Error Field was present in the plasma. The \( m = 2, n = 1 \) harmonic of the Error Field can be estimated as \( B_{3p} = 1.5 \times 10^{-4} \) T at the plasma boundary [15, 16].

The space structure of the MHD mode is measured with a set of poloidal magnetic field sensors located at the inner side of the vacuum vessel wall. The observed \( m = 2 \) mode rotation irregularity has a time-scale comparable with the period of the oscillations. In this case instead of the standard spectral processing of the MHD signals we calculate the in-

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**Fig. 1.** Waveforms of discharge parameters: toroidal magnetic field at the plasma axis, \( B_T \); plasma current, \( I_P \); loop voltage, \( U_P \); line-average plasma density, \( n_e \); electron temperature at plasma axis, \( T_e \); measured by the plasma emission at electron cyclotron frequency; \( m = 2 \) mode amplitude, \( \text{Ampl.} B_n \); and instantaneous frequency, \( \Omega/2\pi \). The vertical dashed lines in two bottom panels show the time interval presented in Fig. 2

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stantaneous value of the mode frequency as the time derivative of the spatial phase of the magnetic perturbation. The processing of the magnetic sensor signals includes the integration taking account of the response time of the sensors and the decomposition of the magnetic field perturbation into the set of spatial Fourier harmonics with different poloidal, \( m \), and toroidal, \( n \), numbers. For each harmonic with certain \( m \) and \( n \) numbers the poloidal magnetic field perturbation at the radial position of the magnetic sensors is

\[
B_0(\theta, \varphi, t) = B_{0C}(t)\cos(m\theta - n\varphi) + B_{0S}(t)\sin(m\theta - n\varphi),
\]

where \( \varphi \) and \( \theta \) are the toroidal and poloidal angles respectively, \( B_{0C}(t) \) and \( B_{0S}(t) \) are the cosine and sine components of the measured harmonic of the magnetic perturbation. The amplitude of the harmonic is

\[
\text{Ampl. } B_0(t) = \sqrt{B_{0C}^2(t) + B_{0S}^2(t)}.
\]

The space phase of this harmonic is defined as \( \Phi(t) = \text{arctan}[B_{0S}(t)/B_{0C}(t)] \) and the instantaneous value of the mode frequency is \( \Omega(t) = d\Phi/dt \). The rotation irregularity factor is defined as \( \Delta\Omega(2 \langle \Omega \rangle) \). In this formula \( \Delta\Omega \) is the difference between maximum and minimum values of \( \Omega \) over the oscillation period and \( \langle \Omega \rangle \) is the half sum of these values.

**EXPERIMENTAL RESULTS**

In this experiment the attention was paid to the dynamics of the \( m = 2 \) mode with relatively small amplitude. The experimental waveforms of the \( m = 2 \) mode signals in the case of the mode amplitude non-monotonic variation in time are shown in Fig. 2. In the figure, one can see that the amplitude variations are followed by changes in the character of the mode rotation. If the amplitude exceeds the level of about \( \text{Ampl. } B_0 = 1.5 \cdot 10^{-4} \text{T} \) the mode is in the state of the irregular rotation. With a decrease of the amplitude lower than this threshold value the mode rotation pauses and recovers when the amplitude surpasses this level again. The observed behavior of the mode is consistent with the assumption made in the Introduction. In the conditions of our experiment the observed threshold value of the amplitude corresponds to the calculated magnetic island width \( W = 0.03 \text{ m} \). Though in this paper we deal with relatively small magnetic islands, this width exceeds by an order of magnitude the level of transition to the Rutherford non-linear regime of the tearing mode [17].

**NUMERICAL MODEL**

The TEAR code [13, 14] is based on the non-linear Rutherford model [17—20] of the tearing mode. The effects of the plasma rotation and RMP are taken into account. As in [21, 22], in cylindrical approximation the equations

\[
\begin{align*}
\frac{\partial}{\partial r} \left( r \frac{\partial \Psi_c}{\partial r} \right) - \left( m^2 + \frac{\mu_0 R}{B_t} \frac{\partial j/\partial r}{\mu - n/m} \right) \Psi_c + \mu_0 \sum_i r_{ic} \delta(r - r_i) &= 0, \\
\frac{\partial}{\partial r} \left( r \frac{\partial \Psi_s}{\partial r} \right) - \left( m^2 + \frac{\mu_0 R}{B_t} \frac{\partial j/\partial r}{\mu - n/m} \right) \Psi_s + \mu_0 \sum_i r_{is} \delta(r - r_i) &= 0
\end{align*}
\]
are used for the calculation of the radial distribution of the cosine, $\Psi_c$, and sine, $\Psi_s$, components of the helical magnetic flux perturbation:

$$\Psi = \Psi_c(r, t)\cos(m\theta - n\varphi) + \Psi_s(r, t)\sin(m\theta - n\varphi).$$

(3)

The poloidal and radial components of the magnetic field perturbation are $B_0 = -\partial\Psi/\partial r$ and $B_r = (1/r)\partial\Psi/\partial \theta$. We assume that $R \gg a$, $m \geq n$.

The solution of (1), (2) satisfies the boundary conditions $\Psi_{C,S}(0) = \Psi_{C,S}(b) = 0$, where $b = 0.5 \text{ m}$ is the effective radius of the perfectly conducting outer wall. In (1) and (2), $j(r)$ is the unperturbed part of the plasma current density, $\mu(r) = 1/\sigma(r)$, $q(r)$ is the safety factor, $\mu_0$ is the magnetic permeability of vacuum. According to calculations for the conditions of our experiment, the current in the vacuum vessel wall did not noticeably affect the dynamics of the MHD mode under consideration.

In this paper we assume that the RMP is produced by two helical currents at different radii outside the plasma, including the current, $i_{1C, S}$, which makes the permanent Error Field and the current with the surface density

$$i_{2C, S} = -\sigma h \frac{d\Psi_{C,S}(r_{VV})}{dt}$$

(5)

generated in the resistive vacuum vessel wall at $r = r_{VV}$ due to the magnetic flux variations in time. In the formula (5), $h = 0.3 \times 10^{-3} \text{ m}$ is the effective thickness of the vacuum vessel wall and $\sigma$ is the stainless-steel wall conductivity. According to calculations for the conditions of our experiment, the current in the vacuum vessel wall did not noticeably affect the dynamics of the MHD mode under consideration.

The solution of the equations (1), (2) represent the superposition

$$\Psi_{C,S}(r) = \Psi_{0C,S}(r) + \Psi_{iC,S}(r)$$

(6)

of the solution $\Psi_{0C,S}(r)$ of homogeneous equations with $i_{kC, S} = 0$ and partial solutions $\Psi_{iC,S}(r)$ of the non-homogeneous equations with $i_{kC, S} \neq 0$ satisfying the boundary conditions $\Psi_{iC,S}(r_S + W/2) = \Psi_{iC,S}(b) = 0$. In this formula, $W = 4 \sqrt{\frac{R}{r_S B_r} \left[ \frac{d\mu}{dr} \right]_{r_S}}$ is the width of the magnetic island, $r_S$ is the radius of the magnetic surface on which $\mu(r_S) = n/m$. The stability index of the tearing mode consists of the axisymmetric part $\Delta'_0(W)$ independent on the external helical current and the part $\Delta'_{iC,S}(W)$ proportional to the components of the external helical current:

$$\Delta'_{C, S}(W) = \frac{\Psi'_{C,S}(r_S + W/2) - \Psi'_{C,S}(r_S - W/2)}{\Psi_{C,S}(r_S)} = \Delta'_0(W) + \Delta'_{iC, S}(W),$$

(7)

where

$$\Delta'_0(W) = \frac{\Psi'_{0C}(r_S + W/2) - \Psi'_{0C}(r_S - W/2)}{\Psi_{0C}(r_S)} = \frac{\Psi'_{0S}(r_S + W/2) - \Psi'_{0S}(r_S - W/2)}{\Psi_{0S}(r_S)},$$

(8)

$$\Delta'_{iC, S}(W) = \frac{\Psi'_{iC,S}(r_S + W/2)}{\Psi_{iC,S}(r_S)}. \tag{9}$$

In (7), (8), (9) $\Psi'$ denotes $d\Psi/dr$. 
The time evolution of the cosine and sine components of the magnetic flux perturbation $\Psi_{C, S}$ at the radius $r = r_s$ is described by the modified Rutherford equations [2, 23, 24]. The mode rotation and variations of tearing mode stability index components $\Delta'_C$ and $\Delta'_S$ under influence of RMP is taken into account as in [20—22]. It has been theoretically justified in [21]. These equations are:

\[
\frac{d \Psi_C}{dt} = \pi d^2 \omega_R \left[ \Delta'_0 + \beta_p \left( \Delta'_B - \Delta'_{GGJ} - \Delta'_{pol} \right) \right] \Psi_C - \Omega_{nat} \Psi_S, \tag{10}
\]

\[
\frac{d \Psi_S}{dt} = \pi d^2 \omega_R \left[ \Delta'_0 + \beta_p \left( \Delta'_B - \Delta'_{GGJ} - \Delta'_{pol} \right) \right] \Psi_S + \Omega_{nat} \Psi_C, \tag{11}
\]

where $\omega_R = 1/\tau_R$ is the inverse resistive time, $\tau_R = \mu_0 \rho^2 / \eta$, $\eta$ is the plasma resistivity and $\beta_p$ is the plasma pressure normalized by the poloidal magnetic field pressure at $r = r_s$. In (10), (11) the terms $\Delta'_B$, $\Delta'_{GGJ}$ and $\Delta'_{pol}$ are the bootstrap, curvature and polarization neoclassical terms respectively (see [2, 23, 24]). Under the corresponding to the experiment quasi-stationary condition, $\frac{d}{dW} \left[ \Delta'_0 + \beta_p \left( \Delta'_B - \Delta'_{GGJ} - \Delta'_{pol} \right) \right] < 0$, the magnetic island width $W$ oscillate around the so-called saturation level $W_{sat}$, that is determined by the condition $\Delta'_0 + \beta_p \left( \Delta'_B - \Delta'_{GGJ} - \Delta'_{pol} \right) = 0$. In the equations (10), (11), $\Omega_{nat} = mV_\theta r_s - nV_\theta R - \Omega_e$, is the natural frequency depending on the poloidal, $V_\theta$, and the toroidal, $V_\phi$, rotation velocities of the resonant plasma layer $r_s - W/2 \leq r \leq r_s + W/2$, as well as on the frequency of the electron diamagnetic drift, $\Omega_e$, [6, 7, 12, 20, 25, 26].

In the TEAR code, the equations of the resonant layer angular motion in the toroidal and poloidal directions are used to calculate the $V_\theta$ and $V_\phi$ time variations due to the interaction between the RMP and magnetic island structure (see [14]):

\[
\frac{I_\phi}{R} \frac{dV_\phi}{dt} = T_{EM, \phi} - T_{F, \phi}, \tag{12}
\]

\[
\frac{I_0}{r_s} \frac{dV_\theta}{dt} = T_{EM, \theta} - T_{F, \theta}. \tag{13}
\]

In the equations (12) and (13), $I_\phi$ and $I_0$ are the toroidal and poloidal moments of inertia of the plasma resonant layer, $T_{EM, \phi}$ and $T_{EM, \theta}$ are the electromagnetic torques, $T_{F, \phi}$ and $T_{F, \theta}$ are the friction torques. The electromagnetic torque is calculated as the reaction to the torque applied to the conductors with the external helical currents from the magnetic field perturbation originated by the tearing mode. The toroidal and poloidal friction torques are supposed to be proportional to the plasma toroidal and poloidal viscosity and to the differences between the corresponding components of the plasma resonant layer velocity, $V_\phi$, $V_\theta$, and the plasma intrinsic rotation velocity, $V_\phi$, $V_\theta$, outside the resonant plasma layer: $T_{F, \phi} = 8 \pi^2 r_s^2 R F_\phi (V_\phi - V_\phi)$ and $T_{F, \theta} = 8 \pi^2 r_s^2 RF_\theta (V_\theta - V_\theta)$. In these formulae, $F_\phi$ and $F_\theta$ are the toroidal and poloidal components of the friction force normalized to 1 m$^2$ of the resonant layer area and to 1 m/s difference between the velocities of the resonant layer and of the intrinsic rotation of the plasma outside the resonant layer.
RESULTS OF NUMERICAL MODELLING

The result of the simulation with the TEAR code of the experiment on the \( m = 2, n = 1 \) mode locking is shown in Fig. 3. In the calculations, the radial profile of the unperturbed plasma current density was specified as \( j_0(r) = J_0[1 - (r/a)^2]^l \), where \( J_0 \) is the current density at the plasma axis, \( l = [q(a)/q(0)] - 1 \). It was assumed that \( q(0) = 1 \). The value of \( q(a) = 2.7 \) corresponded to the experimental conditions with account of toroidal geometry. Though the neoclassical effects in the modified Rutherford equations are incorporated in the model, they do not play an appreciable role in the results of calculations for the conditions of our experiment. The permanent value of plasma intrinsic frequency value \( \Omega_{nat}/2\pi = (mV_{th}/r_a - nV_{a}/R - \Omega_0)/2\pi = 1.4 \text{ kHz} \), that produced the best fit with the experimental data, was used in the calculations. The values of the inverse resistive time, \( \omega_\phi = 240 \text{ s}^{-1} \), and the normalized friction forces \( F_\phi = 2.7 \times 10^5 \text{ Ns}^{-1}\text{m}^{-2} \) and \( F_\theta = 1.6 \times 10^4 \text{ Ns}^{-1}\text{m}^{-2} \) were obtained from the condition of the best fit between the modelling results and the experimental data on the mode rotation irregularity presented in [14]. A linear decrease of the saturated island width, \( W_{sat} \), to the zero value and a subsequent linear rise of \( W_{sat} \) was preset in the calculations to simulate the non-monotonic variation of the mode amplitude. A time-dependent artificial shift along the ordinate axis of the \( \Delta_0 + \beta_p \left( \Delta_{BS} - \Delta_{GGJ} - \Delta_{pol} \right) \) dependence on \( W \) was used for this purpose.

One can see in Fig. 3 that the used numerical model in general features correctly describes the experimental data. The mode rotation stops after the decrease of the amplitude lower than a certain threshold value. The rotation recovers when the growing amplitude exceeds this level again. In the conditions under consideration, the variations of the calculated natural frequency \( \Omega_{nat} \) are small and do not explain the pause of mode rotation. The natural frequency variations increase with the rise of the mode amplitude.

**DISCUSSION**

The explanation of the mode-locking effect under consideration follows from the equations (10), (11). Multiplying the equation (10) by \( \Psi_S \), the equation (11) by \( \Psi_C \) and calculating the difference between these equations, one can obtain the instantaneous value of the mode angular frequency:

\[
\Omega = \frac{d}{dr} \left( \frac{\Psi_S}{\Psi_C} \right) = \Omega_{nat} - \frac{16\pi\omega_\phi a^2 R}{r_a B_r |d\mu/dr|} \frac{\sqrt{(\Psi_S')^2 + (\Psi_C')^2}}{W^2} \sin \left( \Phi_\psi - \Phi_i \right),
\]

where \( \Phi_\psi - \Phi_i \) is the difference between the angular phases of the tearing mode \( \Phi_\psi = \arctan \frac{\Psi_S}{\Psi_C} \) and RMP.
Φᵢ = arctan \( \frac{Ψ_{i,s}}{Ψ_{i,c}} \). The angular frequency depends on the superposition of the plasma resonant layer velocity and the velocity of the mode with respect to the plasma resonant layer. The resonant plasma layer rotation is determined by the first term, Ω_{nat}, in the right-hand-side of (14). The variations of the mode rotation with respect to the plasma resonant layer are described by the second term in the right-hand-side of (14). These variations take place periodically along with the rotation of the mode relating to the RMP. The range of the second term variations is proportional to the RMP value and inversely proportional to \( W^3 \). It increases with the reduction of the magnetic island width. In the case of sufficiently small island width and appropriate phase difference, \( Φ_{Ψ} - Φ_{i} \) the condition of the mode locking \( Ω = 0 \) takes place. It should be noted that second term in the right-hand-side of (14) does not depend on \( Δ_0 \) and on the neoclassical terms in the modified Rutherford equations. As it can be estimated from (14), the mode locking occurs when the magnetic island width is less than the threshold value:

\[
\frac{W}{a} \approx 4 \left( \frac{R}{a} \frac{\omega_{a}}{\Omega_{nat}} \frac{B_{EF}}{B_{T}} \right)^{\frac{1}{3}}. 
\]

This threshold value decreases with reduction of the Error Field because the island width is placed in the denominator of the second term in the right-hand-side of (14). It means that even in the case of arbitrary small Error Field, the mode locking will take place for sufficiently small magnetic islands. This is valid for the mode amplitude higher than the amplitude corresponding to transition from the linear regime of the tearing mode to the Rutherford regime. For our experimental conditions, this formula gives the estimation of the magnetic island width \( W = 0.04 \) m that roughly agree with the value obtained from the experiment.

**SUMMARY**

The presented experimental observation confirms the assumption that the rotation of the mode should stop in the case of sufficiently small amplitude lower than a certain threshold value. The mode rotation recovers when the amplitude exceeds the threshold value. The explanation of the small amplitude mode locking can be attributed to the effect of the permanent Error Field on the mode rotation. The numerical simulation of the tearing mode with sufficiently small amplitude in general features correctly describes the experimental data. According to simulation, the influence of the Error Field on the rotation velocity of the resonant plasma layer is not significant in the conditions of our experiment. In these conditions, the variations of the magnetic island rotation velocity cannot be attributed to the variations of the resonant plasma layer velocity. These mode velocity variations take place due to the asymmetric effect of the Error Field on the tearing mode stability index.

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